Part 1 Review for Quiz 5

① $130° \pm 360° \cdot k$

e.g. $-230°, 490°$

② $15 \cdot \frac{180°}{\pi} = \frac{2700°}{\pi}$

③ $\theta_r = \sin^{-1}\left(\frac{1}{5}\right) = 0.201$

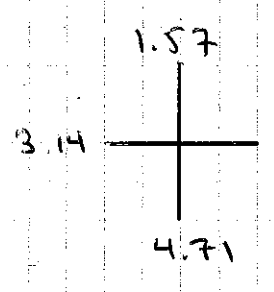
$\sin\theta < 0$, so Q3 or Q4

$\theta = 3.34, 6.08$

④ not defined when $\cos x \leq 0$, which occurs for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

$\therefore f(x)$ is defined for $[0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi]$

⑤ $12 - 2\pi = 5.71$



5.71 is in Q4

$\therefore P(12)$ is in Q4

⑥ $126° \times \frac{\pi}{180°} = \frac{7\pi}{10}$

$s = \theta r = \frac{7\pi}{10}(20) = 14\pi$ or 44 cm

⑦

S	A
T	C

cos is +

csc is − → sin is −

∴ θ is in Q4

⑧ $-\frac{27\pi}{11} \pm 2\pi k$

e.g. $-\frac{5\pi}{11}$, $\frac{17\pi}{11}$

⑨ $x^2 + y^2 = 1$

$\left(\frac{3}{5}\right)^2 + y^2 = 1$

$\frac{9}{25} + y^2 = \frac{25}{25}$

$y^2 = \frac{25 - 9}{25}$

$y^2 = \frac{16}{25}$

$y = \pm\frac{4}{5}$

⑩ $\cos(\theta - \pi)$ is a translation of $\cos\theta$ π units to the right. This produces the same result as $-\cos\theta$ (a reflection in the x-axis).

(11) $y = \frac{1}{\cos x}$

undefined when $\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore$ $y = \sec x$ is undefined for $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Part 2

(1) $72^\circ \times \frac{\pi}{180^\circ} = \frac{2\pi}{5}$

(2) $\sin^2(45^\circ) + \cos^2(60^\circ)$

$= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{2}{4} + \frac{1}{4}$

$= \frac{3}{4}$

(3) $17^\circ \times \frac{\pi}{180^\circ} = \frac{17\pi}{180}$

(4) 1

($\sin^2\theta + \cos^2\theta = 1$ for all values of θ)

(5) both $\cos^2 x$ and $\sin^2 x$ are between 0 and 1

The max value will occur when $\cos^2 x = 1$ and $\sin^2 x = 0$.

$6(1-0) - 3 = 3$

⑥ $\sin^2 x - 3\cos x = 3$

$(1 - \cos^2 x) - 3\cos x = 3$

$1 - \cos^2 x - 3\cos x = 3$

$0 = 3 - 1 + \cos^2 x + 3\cos x$

$\cos^2 x + 3\cos x + 2 = 0$

$(\cos x + 1)(\cos x + 2) = 0$

$\cos x = -1$ $\qquad$ $\cos x = -2$

$x = \pi$ $\qquad$ no solution

⑦ $\tan^2 x = \frac{\sqrt{3}}{3} \cdot \tan x$

$\tan^2 x - \frac{\sqrt{3}}{3} \cdot \tan x = 0$

$\tan x \left(\tan x - \frac{\sqrt{3}}{3} \right) = 0$

$\tan x = 0$ $\qquad$ $\tan x = \frac{\sqrt{3}}{3}$

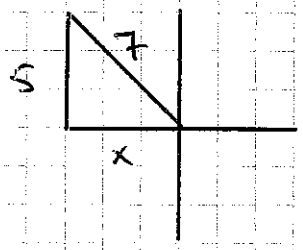
$x = 0, \pi, 2\pi \ldots$ $\qquad$ $x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = K\pi$

$x = \frac{\pi}{6} + 2\pi K, \frac{7\pi}{6} + 2\pi K$

$K \in \mathbb{Z}$

(8) sin is + , cos is − ⟩ Q2



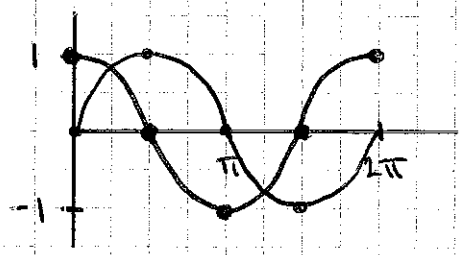
$x^2 + 5^2 = 7^2$

$x = -\sqrt{24}$

$\cot\theta = \frac{adj}{opp}$

$\cot\theta = -\frac{\sqrt{24}}{5}$

(9)



$y = \sin\theta$

$y = \cos\theta$

$\sin\theta < \cos\theta$ from 0 to $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ to 2π

$\therefore \left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

(10) $\tan 3\theta = -1$

$0 \le 3\theta \le 6\pi$

$3\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}, \frac{19\pi}{4}, \frac{23\pi}{4}$

$\theta = \frac{3\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{15\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

(11) $\sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = \frac{\pi}{3} + 2\pi k, \frac{2\pi}{3} + 2\pi k$

$\theta = \frac{\pi}{6} + \pi k, \frac{\pi}{3} + \pi k$

Part 3

① $\frac{13\pi}{7} \pm 2\pi k$

e.g. $-\frac{\pi}{7}$, $27\frac{\pi}{7}$

② $3 \cdot \frac{180^\circ}{\pi} = \frac{540^\circ}{\pi}$

③ $\theta_r = \cos^{-1}\left(\frac{1}{7}\right) = 1.43$

cos is −, so QII, QIII

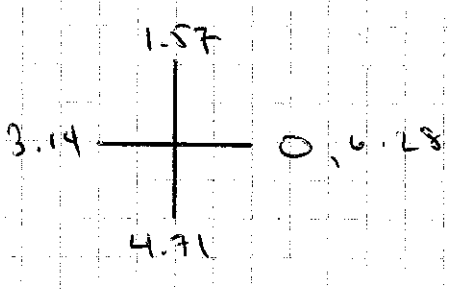
$\theta = 1.71, 4.57$

④ undefined when sin x is negative (π to 2π)

∴ f(x) is defined for $[0, \pi]$

⑤ $18 - 2\pi = 11.72$

$11.72 - 2\pi = 5.43$



5.43 is in QIV

∴ P(18) is in QIV

⑥ $80^\circ \times \frac{\pi}{180^\circ} = \frac{4\pi}{9}$

$s = \theta r = \frac{4\pi}{9}(5) = \frac{20\pi}{9}$ or 6.98 cm

(7)

S	A
T	C

$\sin +$
$\tan -$
$\Big\rangle \therefore Q2$

(8) $7\frac{\pi}{9} \pm 2\pi k$

e.g. $-11\frac{\pi}{9}$, $25\frac{\pi}{9}$

(9) $x^2 + y^2 = 1$

$x^2 + \left(\frac{-5}{13}\right)^2 = 1$

$x^2 + \frac{25}{169} = \frac{169}{169}$

$x^2 = \frac{169-25}{169}$

$x = \pm\sqrt{\frac{144}{169}}$

$x = \pm\frac{12}{13}$

(10) $\sin\left(\theta - \frac{\pi}{2}\right)$ is $\sin\theta$ translated $\frac{\pi}{2}$ right. This produces the same result as $-\cos\theta$ (a reflection of $\cos\theta$ in the x-axis).

(11) undefined for $\sin x = 0$ $\quad x = 0, \pi, 2\pi$

$\therefore$ $y = \csc x$ is undefined for $x = 0, \pi, 2\pi$

Part 4

① $12^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{15}$

② $\sin^2(60^\circ) + \cos^2(60^\circ)$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

③ $9^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{20}$

④ 1 ($\cos^2\theta + \sin^2\theta = 1$ for all θ)

⑤ $\cos^2 x$ and $\sin^2 x$ are both between 0 and 1

∴ max value occurs when $\cos^2 x = 1$ and $\sin^2 x = 0$

$2(1-0) - 4 = -2$

⑥ $\cos^2 x - 3\sin x = 3$

$1 - \sin^2 x - 3\sin x = 3$

$\sin^2 x + 3\sin x + 3 - 1 = 0$

$\sin^2 x + 3\sin x + 2 = 0$

$(\sin x + 1)(\sin x + 2) = 0$

$\sin x = -1 \quad \sin x = -2$

$x = \frac{3\pi}{2}$ no solution

⑦ $\tan^2 x = \sqrt{3}\tan x$

$\tan^2 x - \sqrt{3}\tan x = 0$

$\tan x(\tan x - \sqrt{3}) = 0$

$\tan x = 0$ $\quad$ $\tan x = \sqrt{3}$

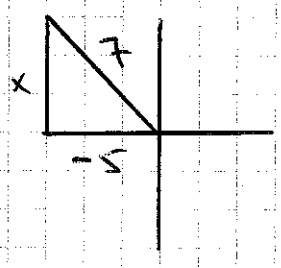
$x = 0, \pi, 2\pi \ldots$ $\quad$ $x = \frac{\pi}{3}, \frac{4\pi}{3}$

$x = K\pi$

$x = \frac{\pi}{3} + 2\pi K, \frac{4\pi}{3} + 2\pi K$

$K \in \mathbb{Z}$

⑧ cos is $-$, tan is $-$ $\Rightarrow$ Q2

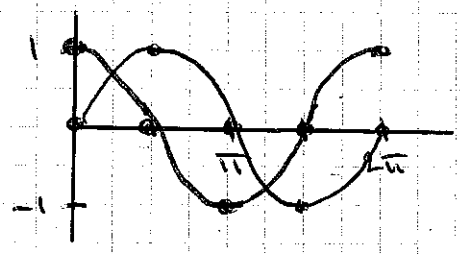


$x^2 + (-5)^2 = 7^2$

$x = \sqrt{24}$

$\sin\theta = \frac{\sqrt{24}}{7}$

$\csc\theta = \frac{7}{\sqrt{24}}$

⑨ 

$y = \sin\theta$

$y = \cos\theta$

$\sin\theta > \cos\theta$ for $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

(10) $2\cos 2\theta - 1 = 0$ $\quad 0 \le 2\theta \le 4\pi$

$2\cos 2\theta = 1$

$\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(11) $\cot 2\theta = \sqrt{3}$

$\tan 2\theta = \frac{1}{\sqrt{3}}$

$2\theta = \frac{\pi}{6} + 2\pi k, \frac{7\pi}{6} + 2\pi k$

$\theta = \frac{\pi}{12} + \pi k, \frac{7\pi}{12} + \pi k$